Graphing Quadratic and Polynomial Functions

Algebra 2

Chapter 2

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

Objectives:

- Graph quadratic functions in standard form.
- Apply transformations to quadratic functions.
- Write a quadratic function in standard form given its graph.

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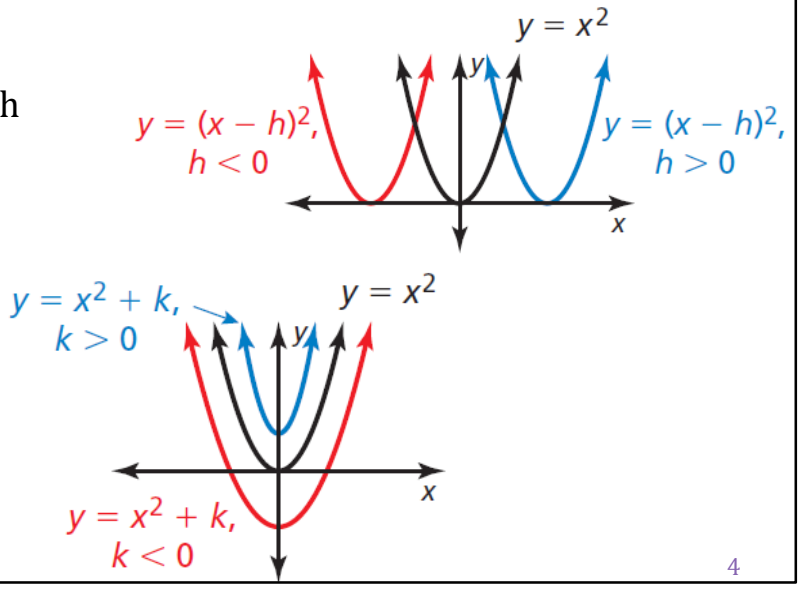
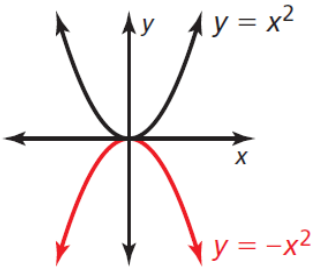
2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

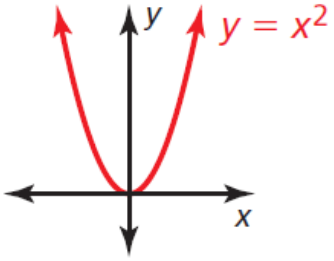
- **Translation**
  - Moves or shifts graph
  - $y = (x - h)^2 + k$
    - h moves right
    - k moves up



2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

- **Reflection**
  - Flips graph over line
  - $y = -(x^2) = -x^2$ reflects over x-axis
  - $y = (-x)^2 = x^2$ reflects over y-axis





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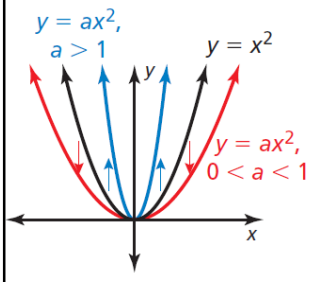
2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)



- **Stretch/Shrink**
  - Stretches or shrinks graph
  - $y = ax^2$
    - a vertical stretch

2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

- Describe the transformation of $f(x) = x^2$ represented by $g(x) = (x - 1)^2 + 2$.

- Describe the transformation of $f(x) = x^2$ represented by $g(x) = \left(\frac{1}{4}x\right)^2 - 2$

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The graph of *g* is a translation 1 unit right and 2 units up of the graph of *f*.

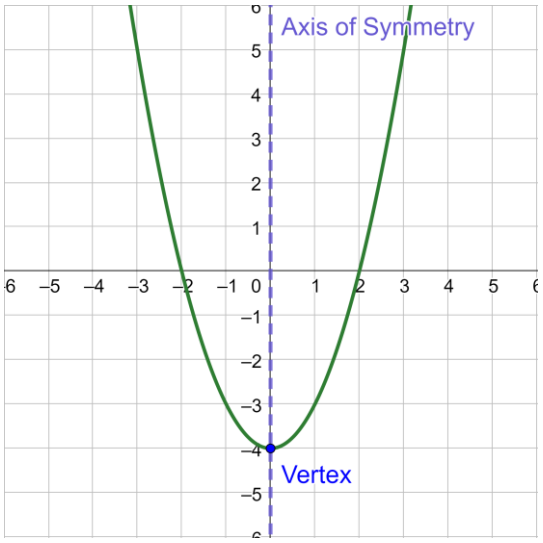
The graph of *g* is a horizontal stretch by a factor of 4, followed by a translation 2 units down of the graph of *f*.

2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

- **Properties of Quadratic Functions in Standard Form (Vertex Form)**

$$f(x) = a(x - h)^2 + k$$

- Vertex is (h, k).
- Wideness of parabola
  - If $|a| > 1$, then it looks narrower than $y = x^2$
  - If $0 < |a| < 1$, then it looks wider than $y = x^2$
- Opens Up/Down
  - If $a > 0$, the parabola opens up.
  - If $a < 0$, the parabola opens down.



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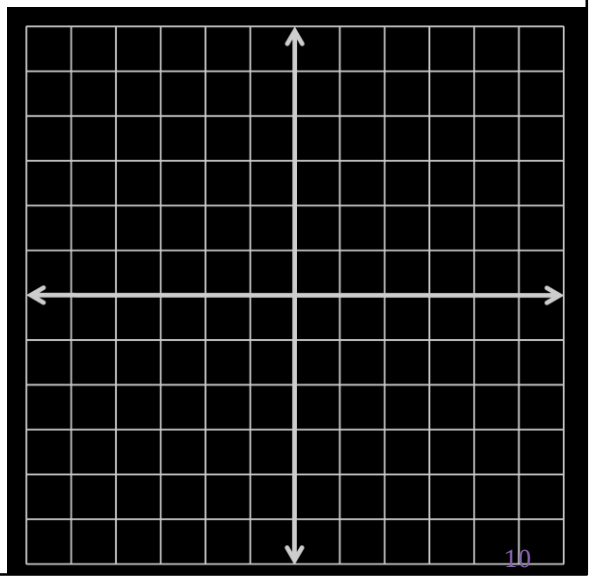
2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

Graph a Quadratic Function

1. Find the vertex. In standard form, the vertex is (h, k).
2. Create a table of values with the vertex in the center.
3. Plot the points from the table of values. At least five points are required.
4. Draw a curve through the points.

2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

- Graph $f(x) = -x^2 + 3$

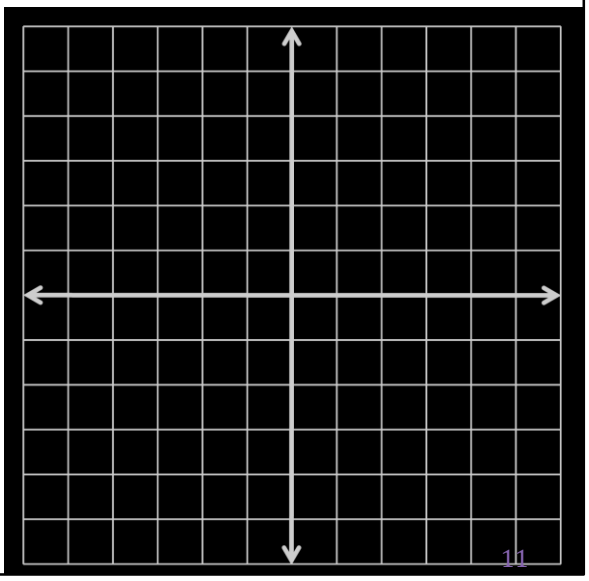


Vertex (0, 3)

$a = -1$

2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

- Graph $f(x) = \frac{1}{2}(x+1)^2 - 2$



Vertex $(-1, -2)$

$$a = \frac{1}{2}$$

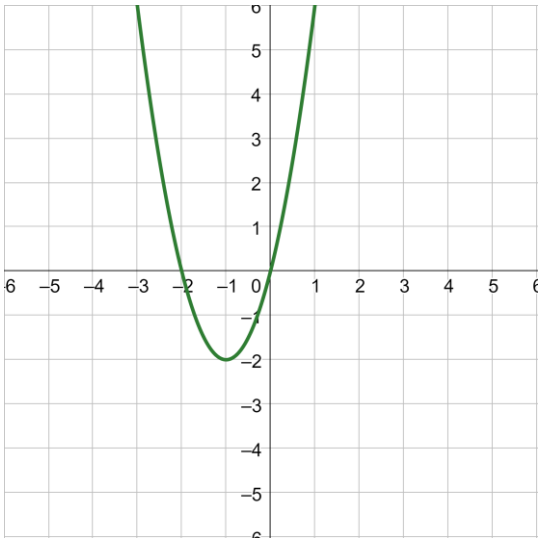
2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

Find a Quadratic Model

1. To find a quadratic model given vertex and another point,
2. Substitute the vertex into standard form, $f(x) = a(x - h)^2 + k$.
3. Substitute the other point for x and y.
4. Solve for a.
5. Write the quadratic function.

2-01 Graph Quadratic Functions in Standard Form (2.1, 2.2)

- Write the quadratic function for the graph.



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Vertex at (−1, −2), so $h = -1$ and $k = -2$.

$$y = a(x - h)^2 + k$$
$$y = a(x + 1)^2 - 2$$

Plug in a point (0, 0).

$$0 = a(0 + 1)^2 - 2$$
$$2 = a$$

Write answer.

$$y = 2(x + 1)^2 - 2$$

2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

Objectives:

- Graph quadratic functions in intercept and general form.
- Find the axis symmetry and vertex of a quadratic function.
- Write a quadratic function in intercept form given its *x*-intercepts.

2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

- **Intercept form**

$$y = a(x - p)(x - q)$$

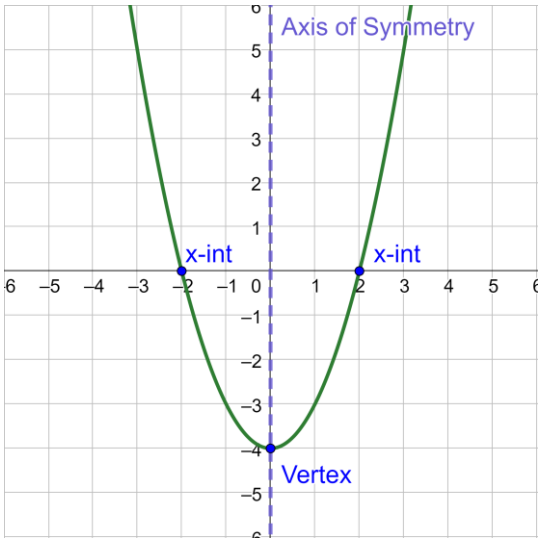
- where p and q are the x-intercepts.

- Axis of symmetry is halfway between the x-intercepts.

$$x = \frac{p + q}{2}$$

- Vertex

$$\left(\frac{p + q}{2}, f\left(\frac{p + q}{2}\right)\right)$$



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2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

- **General Form**

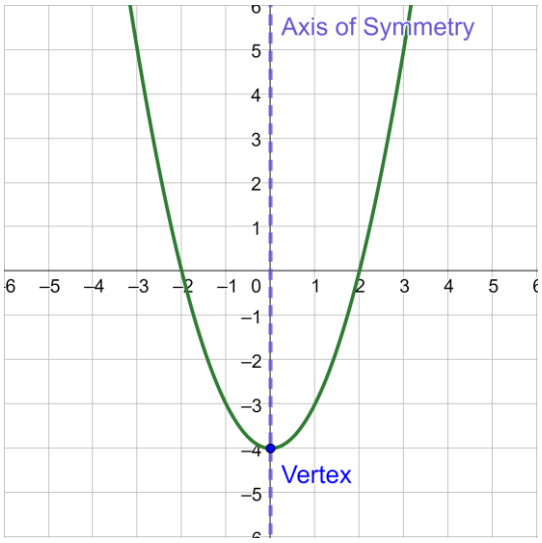
$$y = ax^2 + bx + c$$

- The axis of symmetry is

$$x = -\frac{b}{2a}$$

- Vertex

$$\left(-\frac{b}{2a}, f\left(\frac{b}{2a}\right)\right)$$



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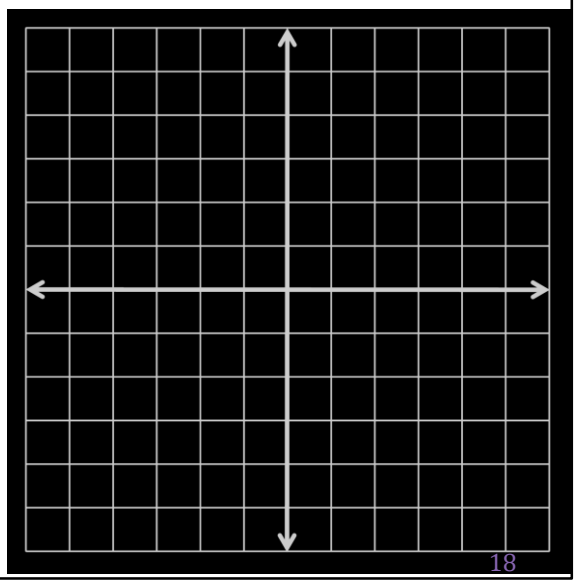
2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

- **Graph a Quadratic Function**

1. Find the axis of symmetry and vertex.
2. Make a table of values using points on either side of the axis of symmetry.
3. Plot the points from the table.
4. Draw the parabola through the points.

2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

- Graph $y = -2(x + 2)(x - 3)$



p = −2 and q = 3

Axis of symmetry

$$x = \frac{p+q}{2} \rightarrow x = \frac{-2+3}{2} \rightarrow x = \frac{1}{2}$$

Vertex (plug in ½ to get y)

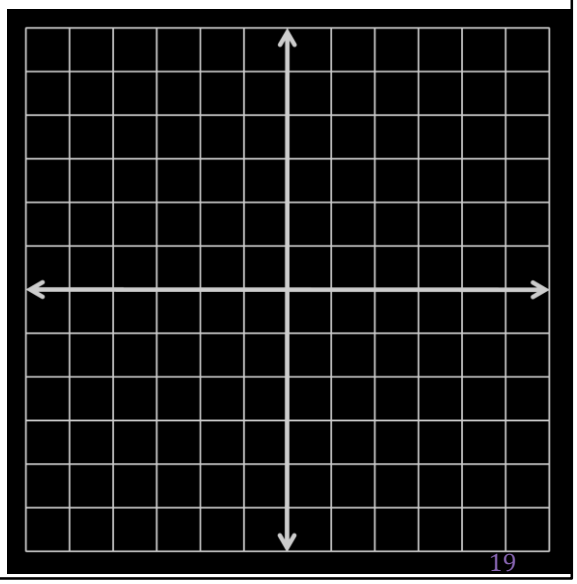
$$y = -2\left(\frac{1}{2} + 2\right)\left(\frac{1}{2} - 3\right) = \frac{25}{2}$$

$$\left(\frac{1}{2}, \frac{25}{2}\right)$$

Make table and graph

2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

- Graph $y = x^2 - 2x - 3$



General form

a = 1, b = −2, c = −3

Axis of symmetry

$$x = \frac{-b}{2a} \rightarrow x = \frac{-(-2)}{2(1)} \rightarrow x = 1$$

Vertex (plug in ½ to get y)

$$y = (1)^2 - 2(1) - 3 = -4$$
$$(1, -4)$$

Make table and graph

2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

Write a Quadratic Function in Intercept Form

1. Find the x-intercepts. These are p and q.
2. Find one other point that the graph passes through. This is (x, y).
3. Substitute the x-intercepts for p and q in intercept form $y = a(x - p)(x - q)$.
4. Substitute the point for x and y.
5. Solve for a.
6. Write the function by substituting p, q, and a into intercept form.

2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

- Write the quadratic function whose x-intercepts are -3 and 7 and passes through $(0, 21)$.

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p = −3 and q = 7

$$y = a(x - p)(x - q)$$
$$y = a(x + 3)(x - 7)$$

Plug in point

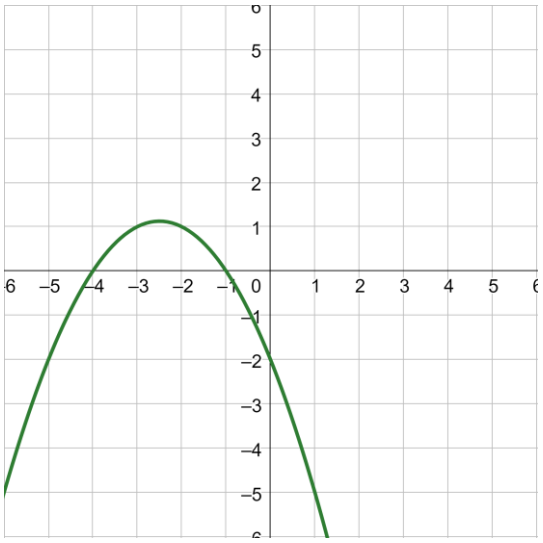
$$21 = a(0 + 3)(0 - 7)$$
$$21 = a(-21)$$
$$-1 = a$$

Write equation

$$y = -(x + 3)(x - 7)$$

2-02 Graph Quadratic Functions in General and Intercept Form (2.2)

- Write the quadratic function given in the graph.



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p = −4 and q = −1
Intercept form

$$y = a(x - p)(x - q)$$
$$y = a(x + 4)(x + 1)$$

Plug in a point (0, −2)

$$-2 = a(0 + 4)(0 + 1)$$
$$-2 = a(4)$$
$$a = -\frac{1}{2}$$

Write equation

$$y = -\frac{1}{2}(x + 4)(x + 1)$$

2-03 Graph Quadratic Inequalities (3.6)

Objectives:

- Graph a quadratic inequality in two dimensions.
- Graph a system of quadratic inequalities in two dimensions.

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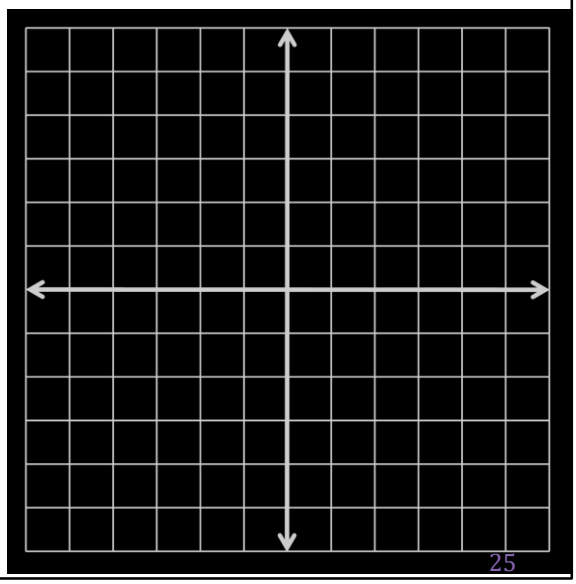
2-03 Graph Quadratic Inequalities (3.6)

- **Graph a Quadratic Inequality in Two Dimensions**

1. Graph the inequality as if it was a function.
2. Decide whether the line is solid or dashed.
   - ≤, =, ≥ → solid line.
   - <, > → dashed line.
3. Decide where to shade.
   - Method 1:
     1. Pick a test point not on the line and substitute it into the original inequality.
     2. If the point is a solution, shade that side of the parabola.
     3. If it is NOT a solution, shade the other side.
   - Method 2:
     1. Solve the inequality for y.
     2. If the inequality is $y >$, shade above the parabola.
     3. If the inequality is $y <$, shade below.

2-03 Graph Quadratic Inequalities (3.6)

- Graph $y \leq \frac{1}{2}(x+2)(x-4)$



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x-intercepts −2, 4

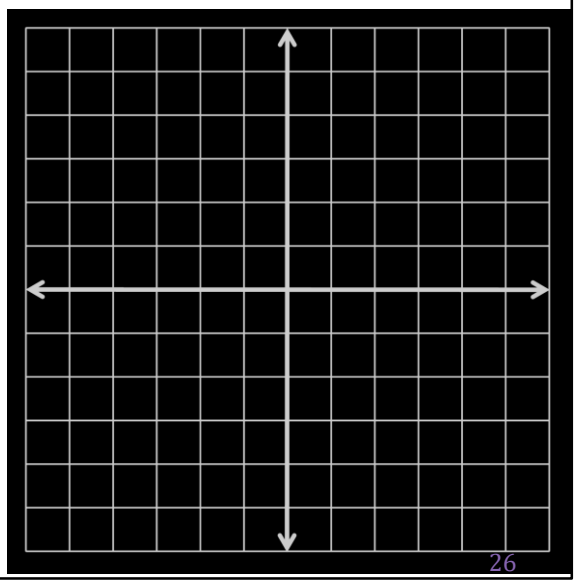
Vertex at $x = \frac{-2+4}{2} = 1$

Solid line

y ≤ 0, shade below

2-03 Graph Quadratic Inequalities (3.6)

- Graph $y > 2(x-2)^2 - 5$



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Vertex at (2, −5)
Dotted line
y > 0, shade above

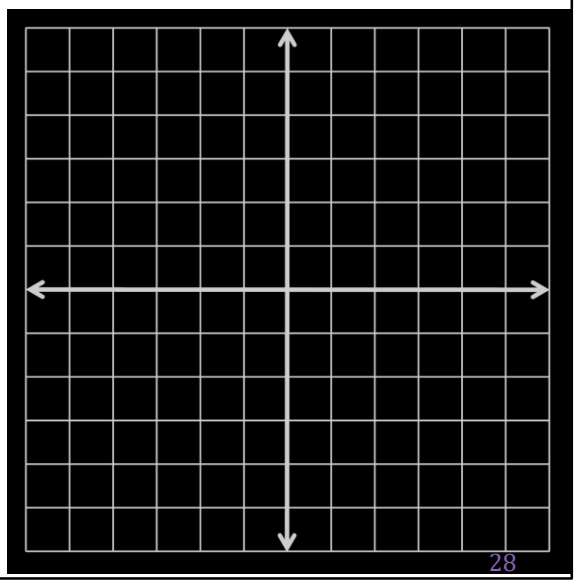
2-03 Graph Quadratic Inequalities (3.6)

Graph a System of Quadratic Inequalities

1. Graph each inequality on the same coordinate plane.
2. The solution is all the points where all the shaded areas overlap.
3. If there is no overlap of the shaded areas, then there is no solution.

2-03 Graph Quadratic Inequalities (3.6)

- Solve $\begin{cases} y \geq x^2 - 4 \\ y \leq -x^2 + 2x + 3 \end{cases}$



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$$y \geq x^2 - 4$$

Vertex at $x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0$

Solid line

y ≥ 0, shade above

$$y \leq -x^2 + 2x + 3$$

Vertex at $x = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$

Solid line

y ≤ 0, shade below

2-04 Graph Polynomial Functions (4.1, 4.8)

Objectives:

- Describe the end behavior of a polynomial functions.
- Graph polynomial functions.
- Find the turning points from a graph.
- Find the x-intercepts from a graph.

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2-04 Graph Polynomial Functions (4.1, 4.8)

- **Polynomial in one variable**
  - Function that has one variable and there are powers of that variable and all the powers are positive
- $4x^3 + 2x^2 + 2x + 5$
- $100x^{1234} - 25x^{345} + 2x + 1$
- $\frac{2}{x}$
- $3xy^2$

Not Polynomials in one variable.

2-04 Graph Polynomial Functions (4.1, 4.8)

- **Degree**
  - Highest power of the variable

- What is the degree?
  - $4x^3 + 2x^2 + 2x + 5$

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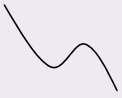
2-04 Graph Polynomial Functions (4.1, 4.8)

- **Types of Polynomial Functions**
- Degree → Type
  - 0 → Constant → $y = 2$
  - 1 → Linear → $y = 2x + 1$
  - 2 → Quadratic → $y = 2x^2 + x - 1$
  - 3 → Cubic → $y = 2x^3 + x^2 + x - 1$
  - 4 → Quartic → $y = 2x^4 + 2x^2 - 1$

2-04 Graph Polynomial Functions (4.1, 4.8)

- **End Behavior**
  - Polynomial functions always go towards ∞ or -∞ at either end of the graph

	Leading Coefficient +	Leading Coefficient -
Even Degree		
Odd Degree		

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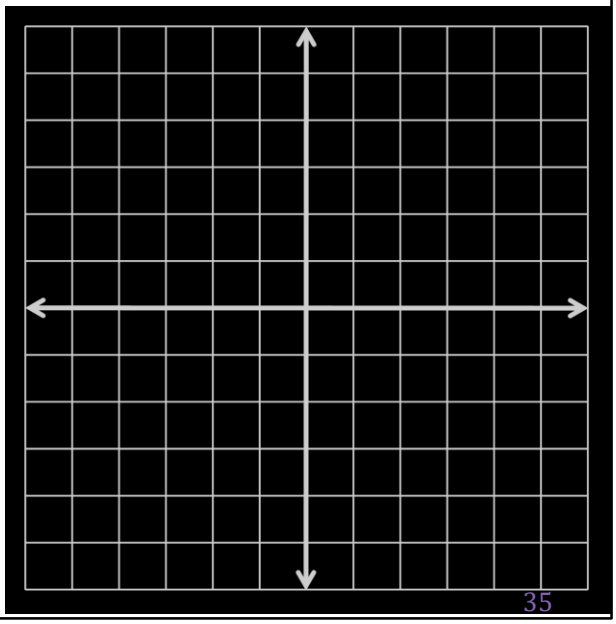
(this is for even degree with positive first term)

2-04 Graph Polynomial Functions (4.1, 4.8)

- **Graphing polynomial functions**
  1. Make a table of values
  2. Plot the points
  3. Make sure the graph matches the appropriate end behavior

2-04 Graph Polynomial Functions (4.1, 4.8)

- Graph $f(x) = x^3 + 2x - 4$



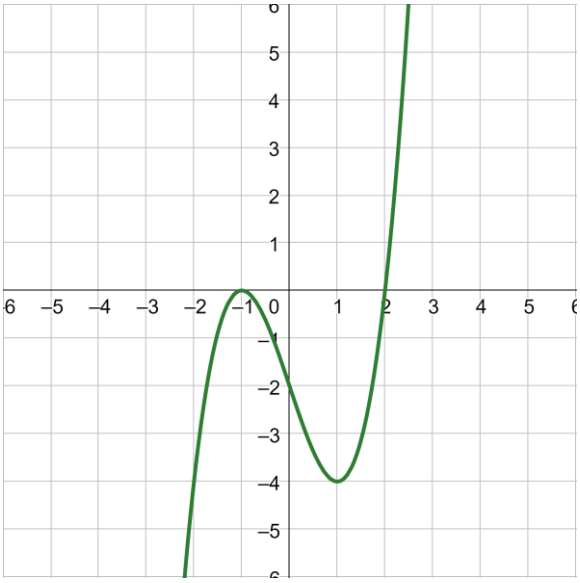
Make table, points are (−3, −37), (−2, −16), (−1, −7), (0, −4), (1, −1), (2, 8), (3, 29)

2-04 Graph Polynomial Functions (4.1, 4.8)

- ***x*-intercepts**
  - Points where the graph crosses the *x*-axis
- **Turning Points**
  - Local Maximum and minimum (turn from going up to down or down to up)
  - The graph of every polynomial function of degree n can have at most $n-1$ turning points.
  - Calculus lets you find the turning points easily.

2-04 Graph Polynomial Functions (4.1, 4.8)

- What are the *x*-intercepts and turning points?



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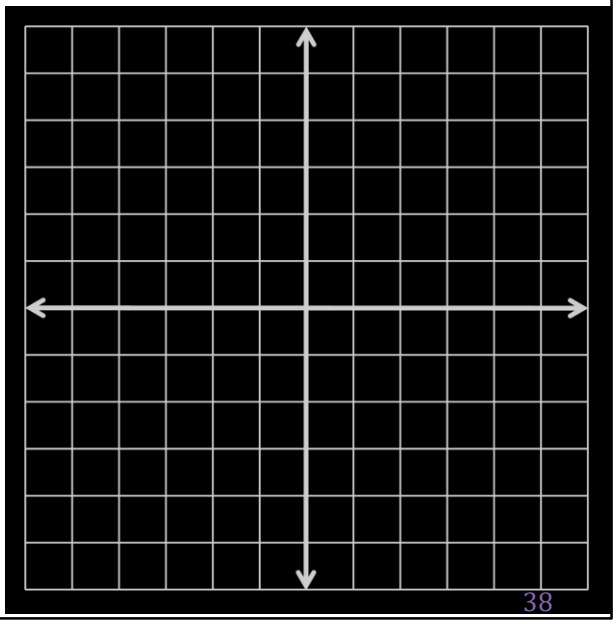
x-ints: (−1, 0), (2, 0)
Max: (−1, 0)
Min: (1, −4)

2-04 Graph Polynomial Functions (4.1, 4.8)

- Graph $f(x) = x^3 - 2x^2 - x + 2$ and estimate the x-intercepts and turning points.



x-ints: (−1, 0), (1, 0), (2, 0)
Max: (−0.22, 2.11)
Min: (1.55, −0.63)

2-05 Write Quadratic and Polynomial Models (4.9)

Objectives:

- Write a polynomial model given the x-intercepts and one other point.
- Write a polynomial model given points on the curve using finite differences and technology.
- Write a best-fitting polynomial model for real-life situations using technology.

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2-05 Write Quadratic and Polynomial Models (4.9)

- **Find a Polynomial Model Given *x*-intercepts**
  1. Write a polynomial model in the form $y = a(x - k_1)(x - k_2)(x - k_3)\ldots$ where there is one factor per x-intercept.
  2. Substitute the x-intercepts for the k's.
  3. Substitute the other point for x and y.
  4. Solve for a.
  5. Write the polynomial function.

2-05 Write Quadratic and Polynomial Models (4.9)

Write a polynomial model with x-intercepts are −2, 1, 3 and (0, 2)

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$$y = a(x+2)(x-1)(x-3)$$

$$2 = a(0+2)(0-1)(0-3) \quad 2 = 6a \quad a = \frac{1}{3}$$

$$y = \frac{1}{3}(x+2)(x-1)(x-3)$$

2-05 Write Quadratic and Polynomial Models (4.9)

Find the Degree of a Polynomial Using Finite Differences

1. Have a table of values with equally spaces *x*-values.
2. Find the differences of successive *y*-values.
3. Find the differences of successive differences from the previous step.
4. Repeat until all the differences in a step are the same number (not zero).
5. The number of levels of differences is the degree of the function.

2-05 Write Quadratic and Polynomial Models (4.9)

- Find the degree of the polynomial passing through (0, 1), (1, 6), (2, 25), (3, 70), (4, 153), (5, 286)

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```
x 0   1   2   3    4    5
y 1   6   25  70   153  286
  \ / \ / \ / \  / \  /
   5   19  45   83   133
    \ / \ / \ /\  /
     14   26   38  50
      \  /\  / \ /
       12   12   12
```

Third order differences, so degree 3

2-05 Write Quadratic and Polynomial Models (4.9)

- **Finding a model given several points**

1. Use finite difference to find the degree.
2. Use either of the following methods.

- **Method 1: Solve a System of Equations by Hand (This lesson uses Method 2)**

a. Write a general polynomial function of the given degree such as $y = ax^3 + bx^2 + cx + d$.
b. Substitute a point for x and y to get an equation where the variables are the coefficients.
c. Substitute another point in the general polynomial for x and y to get a second equation where the variables are the coefficients.
d. Continue substituting points until there the same number of equations as coefficients.
e. Solve the system of equations using something like elimination to find the values of the coefficients.
f. Write the equation by substituting the coefficients into the general polynomial.

- **Method 2**: **Use a Regression on a Graphing Calculator**

2-05 Write Quadratic and Polynomial Models (4.9)

- **Finding Linear Regression on a TI-84**

1. Push STAT and select Edit….
2. Enter the *x*-values in List 1 (L1) and the *y*-values in List 2 (L2).
3. To see the graph of the points
   a. Push Y= and clear any equations.
   b. While still in Y=, go up and highlight Plot1 and press ENTER.
   c. Press ZOOM and select ZoomStat.
4. Push STAT and move over to the CALC menu.
5. Select the type of regression.
6. Make sure the Xlist: is L1, the Ylist: is L2, the FreqList: is blank, and the Store RegEQ: is Y1.
   a. Get Y1 by pressing VARS and select Y-VARS menu.
   b. Select Function….
   c. Select Y1.
7. Press Calculate
8. The calculator will display the equation. To see the graph of the points and line, press GRAPH.

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Note: Older TI graphing calculators do not have the screen in steps 6 and 7. After selecting the LinReg(ax+b), the screen just shows "LinReg(ax+b)". Press ENTER again to see the result. To see the graph, enter the equation into the Y= screen and press GRAPH.

2-05 Write Quadratic and Polynomial Models (4.9)

Finding Linear Regression on a NumWorks graphing calculator

1. On the home screen select Regression.
2. In the Data tab, enter the points.
3. Move to the Graph tab.
4. The default is a linear regression and is displayed at the bottom of the screen. To change the regression type
   a. Press OK.
   b. Select Regression.
   c. Select the desired regression type.

2-05 Write Quadratic and Polynomial Models (4.9)

- Find a polynomial function passing through (1, −2), (2, 2), (3, 12), (4, 28), (5, 50), (6, 78)

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Find finite differences

x	1	2	3	4	5	6
y	−2	2	12	28	50	78
		4	10	16	22	28
			6	6	6	6

Second order differences so degree = 2

Use a calculator or do by hand

$$f(x) = ax^2 + bx + c$$
$$-2 = a(1)^2 + b(1) + c$$
$$2 = a(2)^2 + b(2) + c$$
$$12 = a(3)^2 + b(3) + c$$
$$f(x) = 3x^2 - 5x$$

2-05 Write Quadratic and Polynomial Models (4.9)

- **Best-Fitting Polynomial Models**
- Real-life usually doesn't fit a model exactly, so finite differences don't work.

1. Use a calculator find regressions of several degrees.
2. Choose the one that seems to fit the data the best as shown on the graph.